Gravitational Fields Review

① $\frac{r_H^3}{T_H^2} = \frac{r_E^3}{T_E^2}$

$\frac{r_H^3}{(76)^2} = \frac{(1.49 \times 10^{11})^3}{(1)^2}$

$r_H = \sqrt[3]{(76)^2 (1.49 \times 10^{11})^3}$

$r_H = \boxed{2.67 \times 10^{12} \text{ m}}$

② $\frac{r_s^3}{T_s^2} = \frac{r_m^3}{T_m^2}$

$\frac{r_s^3}{1^2} = \frac{(3.8 \times 10^8)^3}{(27.33)^2}$

$r_s = \sqrt[3]{\frac{(3.8 \times 10^8)^3}{(27.33)^2}}$

$r_s = \boxed{4.19 \times 10^7 \text{ m}}$

③ $F_g = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11})(1.7 \times 10^{-27})(9.1 \times 10^{-31})}{(1 \times 10^{-10})^2}$

$F_g = \boxed{1.03 \times 10^{-47} \text{ N}}$

4. $F_g = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11})(1)(1)}{(1)^2} = \boxed{6.67\times10^{-11}\ N}$

5. $F_g = \frac{GMm}{r^2}$

$1.9\times10^{20} = \frac{(6.67\times10^{-11})(5.98\times10^{24})\,m}{(3.9\times10^{8})^2}$

$m = \boxed{7.25\times10^{22}\ Kg}$

6. $\cancel{m}g = \frac{GM\cancel{m}}{r^2}$

$g = \frac{GM}{r^2}$

Mass x2

Since $g \propto M$, doubling the mass will double g.

$\therefore\ 2\times9.8 = \boxed{19.6\ m/s^2}$

Radius x2

since $g \propto \frac{1}{r^2}$, doubling r will result in $\frac{1}{4}g$.

$\therefore\ \frac{1}{4}\times9.8 = \boxed{2.45\ m/s^2}$

Both

$2\times\frac{1}{4}\times9.8 = \boxed{4.9\ m/s^2}$

7 $v = \frac{2\pi r}{T} = \frac{2\pi(1\times10^7)}{(9.9\times10^3)} = 6346.652$ m/s

$\Sigma F = F_g$

$\frac{\cancel{m} v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$

$M = \frac{rv^2}{G}$

$= \frac{(1\times10^7)(6346.652)^2}{(6.67\times10^{-11})}$

$M = \boxed{6.04\times10^{24}\ Kg}$

8 $\frac{r^3}{T^2} = K$

$r^3 = KT^2$

so $T \propto \sqrt{r^3}$

Substitute 2 for r (because you are doubling the radius).

$\sqrt{2^3} = 2.83$

So the period will be 2.83x longer.

So $2.83 \times 27.33 = \boxed{77.3 \text{ days}}$

(9)

$$\Sigma F = F_g$$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(6.37\times10^{23})}{(3.43\times10^6 + 175\,000)}}$$

$$v = \boxed{3433 \text{ m/s}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(3.43\times10^6 + 175\,000)}{3433}$$

$$T = \boxed{6598 \text{ s}}$$

(10)

$$E_B = -E_{total}$$

$$= -(5\times10^9 + (-6.4\times10^9))$$

$$E_B = \boxed{1.4\times10^9 \text{ J}}$$

(11)

$$0 = \tfrac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r}$$

$$v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2(6.67\times10^{-11})(1.98\times10^{30})}{(6.95\times10^8)}}$$

$$v = \boxed{616\,479 \text{ m/s}}$$

(12) a) $W = \Delta E_g$

$= \left(-\frac{GMm}{r_2}\right) - \left(-\frac{GMm}{r_1}\right)$

$= -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$= -(6.67\times10^{-11})(5.98\times10^{24})(1200)\left[\frac{1}{6.38\times10^{6}} - \frac{1}{(6.38\times10^{6} + 2000\ 000)}\right]$

$W = \boxed{-1.79\times10^{10}\ J}$

b) $\Delta E_k = -\Delta E_g$

$\frac{1}{2}mv_f^2 - 0 = -\Delta E_g$

$\frac{1}{2}(1200)v_f^2 = -(-1.79\times10^{10})$

$v_f = \boxed{5463\ m/s}$

(13) a) $E_g = -\frac{GMm}{r}$

$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(500)}{(6.38\times10^{6} + 200\ 000)}$

$E_g = \boxed{-3.03\times10^{10}\ J}$

13 b) $\Sigma F = F_g$

$$\frac{\cancel{m} v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$E_K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left(\sqrt{\frac{GM}{r}}\right)^2$$

$$E_K = \frac{1}{2} \frac{GMm}{r}$$

$$= \frac{1}{2} \frac{(6.67\times10^{-11})(5.98\times10^{24})(500)}{(6.38\times10^{6} + 200\,000)}$$

$$E_K = \boxed{1.52\times10^{10}\ J}$$

c) $E_B = -E_T$

$$= -(E_K + E_g)$$

$$= -\left(1.52\times10^{10} + (-3.03\times10^{10})\right)$$

$$E_B = \boxed{1.52\times10^{10}\ J}$$

(14) a) $E_g = -\frac{GMm}{r}$

$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(1\times10^{4})}{(1\times10^{10})}$

$E_g = \boxed{-3.99\times10^{8}\ J}$

b) To escape, E_K must exceed E_g

$\therefore\ E_K \boxed{> 3.99\times10^{8}\ J}$

c) $\frac{1}{2}mv^2 = 3.99\times10^{8}$

$\frac{1}{2}(1\times10^{4})v^2 = 3.99\times10^{8}$

$v = \boxed{282\ m/s}$

(15) $E_{surface} = E_{altitude}$

$\frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{r_1} = -\frac{GM\cancel{m}}{r_2}$

$\frac{1}{2}v^2 = \frac{GM}{r_1} - \frac{GM}{r_2}$

$= \frac{(6.67\times10^{-11})(6.7\times10^{22})}{(1.6\times10^{6})} - \frac{(6.67\times10^{-11})(6.7\times10^{22})}{2(1.6\times10^{6})}$

$\frac{1}{2}v^2 = 1396531$

$v = \boxed{1671\ m/s}$